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FRAMEWORK FOR DESIGNING AND IMPROVING NOVEL ORIGAMI PATTERNS USING THE HAMILTONIAN CIRCUIT METHODOLOGY

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ABSTRACT

The ability to compactly fold space-based arrays has seen an increasing demand in the aerospace industry. This work will review the methods which have been developed by various authors to create novel origami patterns which can stow compactly and deploy out to a predictable shape using Hamiltonian Circuits, discuss techniques for modifying and improving the behavior and performance of kirigami patterns created using this method, and then show an example application of this method for creating novel patterns for a particular use case.

Keywords: Origami, Small Satellites, Deployable, Hamiltonian Circuit

NOMENCLATURE

Variables

n Number of sides on the base polygon
N Number of panels in a pattern
M Mobility, or degrees-of-freedom, of a system
j Number of joints in a system
f_i Freedom of a joint
r Number of panels removed from a pattern

Terminology

Pair Two panels which are adjacent to a line of a folding set which intersects a given circuit
Dyad Top and bottom pairs of a folded pattern which define a circuit
Circuit A series of panels which are adjacent to each other, in which each panel is visited exactly once

1. INTRODUCTION

The design of mechanisms which can be deployed to a large surface from a small volume has been a goal of engineers for decades. This application is particularly important to spacecraft. Because sending material to space is so costly relative to terrestrial endeavours, each mission typically has only the equipment required for its specific purpose. The performance of many types of mission-critical equipment is proportional to the amount of surface area that equipment has, such as the power generated by solar panels, the optical capability of telescope arrays, and signal transfer capability of RF antennas. For this reason, many spacecraft incorporate deployable arrays so they can maximize the area of these components in the limited space available on launch, and an important focus of design has become increasing a spacecraft's ratio of deployed surface area to stowed volume to get the largest deployed array from the smallest launch payload. Typically, deployable structures are made out of interlinking panels that fold to a small volume before launch and can be deployed to a larger surface. This has led many to turn to the application of origami and kirigami (introducing cuts into an origami pattern) to aid in the folding and compaction of these arrays.

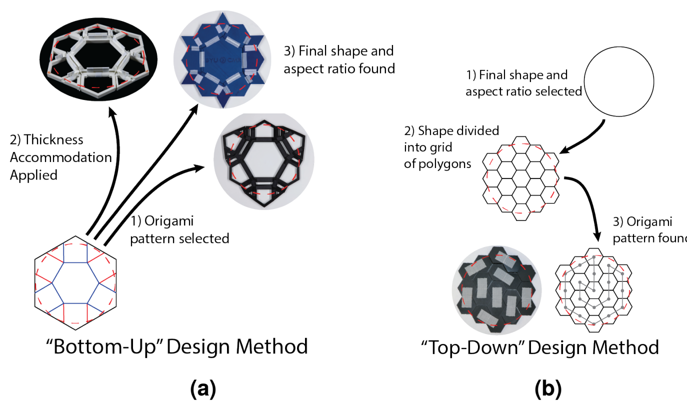

FIGURE 1: DESIGN METHODOLOGIES. A) "BOTTOM-UP" B) "TOP-DOWN".

1.1 Design Methodologies

A common design process using origami begins with a candidate pattern, which is then thickened from the original paper model using a thickness accommodation technique[1], as de-

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scribed by Bolanos et al.[2]. The same origami pattern may result in drastically different designs when incorporating different thickness accommodation patterns, which can make evaluating the final shape of these design more challenging. This "Bottom-Up" process results in various mechanical systems, with varying shapes, degrees of stowed volume efficiency, and deployed area efficiency, as shown in Fig.1 (a). This work builds on a method of a "Top-Down" design approach using Hamiltonian circuits, which allows a designer to choose a final deployed shape and aspect ratio, and then determine a corresponding kirigami pattern, as shown in Fig.1 (b). This approach results in a high deployed area efficiency and stowed volume efficiency by incorporating the hinge-shift thickness accommodation technique. This work also utilizes research done by Yang et al[3–5] to explore the capability of the Hamiltonian circuit method to fold identical thick panels. Other work has explored the design of single degree-of-freedom systems using this methodology such as those described in Yang et al.[6] and Yang et al.[7]. This work will not consider design for a single degree-of-freedom, however, future work could use those concepts to further constrain the designs explored in this work to reduce them even further. Figure1 shows an overview of how each of these design methods works. It should be especially noted how the "Bottom-Up" method results in various deployed shapes which may or may not align with the origami pattern shape originally chosen by the designer. This is contrasted with the "Top-Down" method, in which a kirigami pattern is created for a final shape which can be precisely known beforehand and can align very closely with the desired deployed shape which is chosen by the designer.

This method allows designers the ability to tailor a shape to their specific needs and use-case, as well as providing a robust thickness accommodation method which results in 75% to 100% stowed volume efficiency, depending on the polygon chosen for the base grid. Examples of patterns created with square-based and hexagon-based grids are shown in Fig.2, with the deployed pattern shown on top and the compactly stowed pattern shown on bottom.

This work will review the methods which have been developed by various authors to create novel origami patterns using Hamiltonian Circuits, discuss techniques for modifying and improving the behavior and performance of kirigami patterns created using this method, and then show an example application of this method for creating novel patterns for a particular use case.

2. BACKGROUND

The basis of creating a Hamiltonian circuit was first described by the mathematician William Rowan Hamilton in 1843 when he discovered the system of quaternions, which extend complex numbers to three-dimensional space that can be used in describing certain mechanics [8]. While the theory underlying these principles are thorough and rigorous, the principles themselves allow for simplified applications, such as creating a Hamiltonian circuit. A Hamiltonian circuit (sometimes referred to as a Hamiltonian path or chain), is a concept used in graph theory which connects a series of adjacent points by starting and ending on the same point and visiting each point exactly one time. This concept has been used in computer science applications which deal with

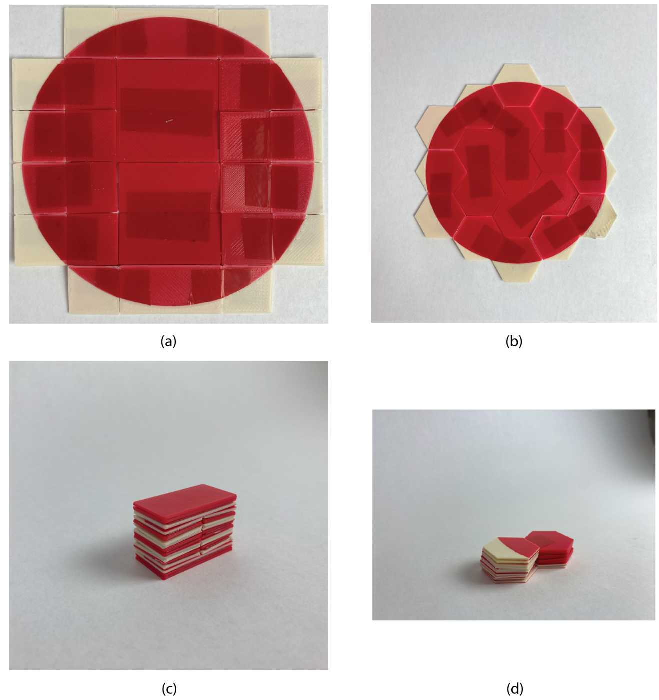


FIGURE 2: SCALE COMPARISON OF PROTOTYPES OF THE 32 PANEL SQUARE PATTERN (LEFT) AND THE 19 PANEL HEXAGON PATTERN (RIGHT) SHOWN DEPLOYED ON THE TOP AND STOWED ON THE BOTTOM.

multiple potential solutions, such as planning routes, scheduling tasks, and designing optimal computational sequences. However, for many of these applications the Hamiltonian circuit method is non-ideal as it is NP-complete, meaning that it may be impossible to solve in polynomial time. The use of Hamiltonian circuits for creating folding circuits in kirigami designs was proposed by Yang et al. [5] for the application of folding rigid panels with uniform thickness. They demonstrated the ability of Hamiltonian circuits to help in the design of the placement of revolute joints throughout a grid of uniform polygons, such that the grid could be folded up and stowed compactly.

The general method of using Hamiltonian circuits to design a kirigami folding pattern is as follows. The designer begins with a grid of uniform polygons, which represent a deployed pattern of individual uniform panels. A desired deployed shape is chosen and imposed on the grid, leaving only the panels required to create the shape. A closed circuit is then found which visits each panel exactly one time. This circuit is then used to determine cut and fold locations within the pattern. This process is shown in Fig.3 as applied to producing an arbitrary pattern, representing the ability of this method to produce unique, unconstrained, and versatile resulting shapes. Determining the folding sequence for the chosen pattern is more difficult and will be detailed hereafter.

A major consideration when creating a pattern is that the pattern should contain an even number of panels. This is because the folding is dependent on a sequence of mountain-valley folds, and patterns with an uneven number of panels would result in folds that did not align at the ends of the circuit. Methods for developing patterns with an odd number of panels are discussed

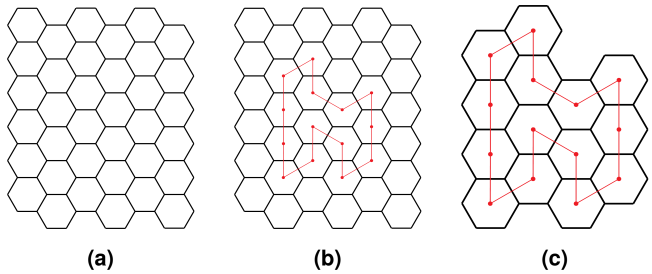


FIGURE 3: EXAMPLE ORIGAMI PATTERN SELECTION USING HAMILTONIAN CIRCUITS. NOTE THAT AN UNUSUAL PATTERN WAS CHOSEN TO ILLUSTRATE HOW A CIRCUIT MAY CONSIST OF A GROUP OF ANY GRID CELLS. A) INITIAL GRID OF HEXAGONS. B) HAMILTONIAN CIRCUIT DRAWN ONTO GRID. B) ORIGAMI PATTERN EXTRACTED FROM GRID.

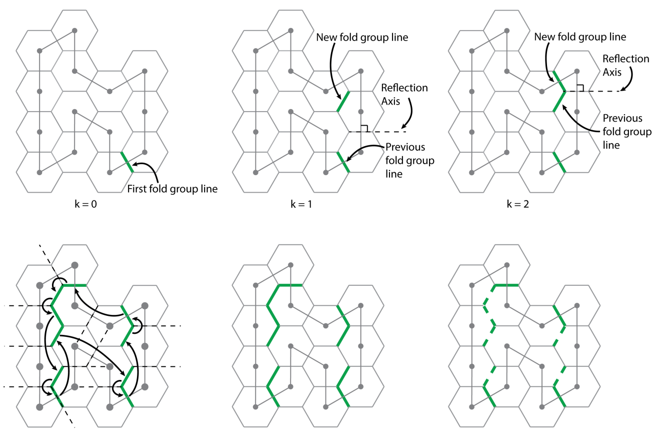


FIGURE 4: REFLECTION SEQUENCE USED TO FIND VALID FOLD GROUPS.

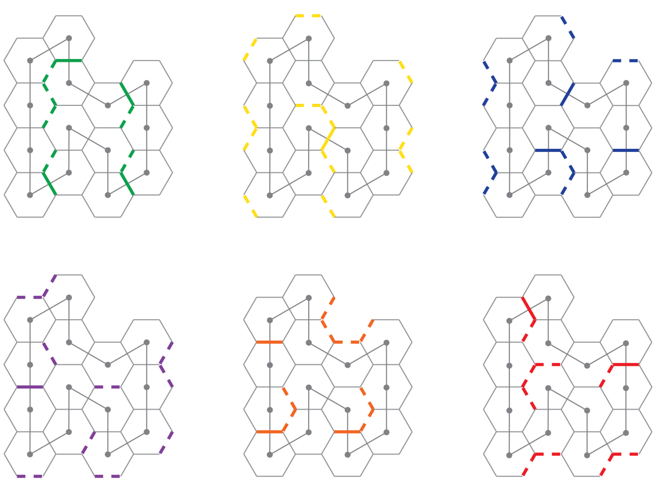


FIGURE 5: UNIQUE FOLD GROUPS FOR SELECTED ORIGAMI PATTERN. NOTE THAT BECAUSE THE PRINCIPAL POLYGON WAS SIX SIDED, THERE ARE SIX UNIQUE FOLD GROUPS.

in Section 5.2. Further considerations for choosing a base grid and resulting pattern will be discussed throughout this work.

It should be noted that combinations of varying polygons may also be used, although it is generally more complicated to illustrate; however, a pattern using both squares and triangles

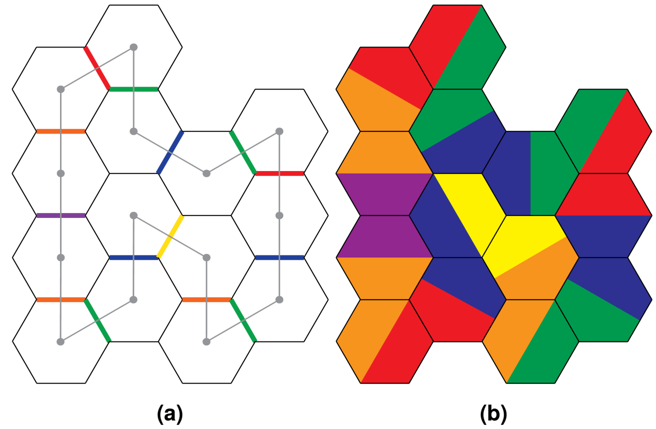


FIGURE 6: FIX THIS FIGURE....RED IS WRONG ON BOTTOM LEFT AND IT MAY BE TOO CONFUSING. ALL VALID FOLDING GROUPS FOR SELECTED ORIGAMI PATTERN. A) FINAL FOLDING DYADS, I.E. CIRCUIT GROUP ELEMENTS THAT INTERSECT HAMILTONIAN CIRCUIT. B) COLOR CHART SHOWING FOLDING GROUPS. ALL DYADS OF HEXAGONS WITH IDENTICAL COLORS ARE VALID FOLDING COMBINATIONS WITH ANY OTHER DYAD OF THE SAME COLOR. NOTE THAT TWO GROUPS, PURPLE AND YELLOW, HAVE ONLY ONE DYAD IN THEIR GROUP, MEANING THAT THEY WILL NOT RESULT IN A VALID FOLDING SEQUENCE.

will be shown in Section 6. Additionally, the polygon chosen for the base grid will determine the maximum volumetric stowage efficiency, as different shapes are able to fill a cuboid volume differently.

3. CREATING SIMPLE PATTERNS

As explained before, creating a Hamiltonian circuit is an NP-complex problem, and no valid solution is guaranteed for an arbitrary set of points, however, by constraining the points to a predetermined grid of identical polygons, the number of solutions increases, even becoming exponentially related to the number of panels in the chosen pattern, or N. The pattern created will also have a folding possibilities proportional to the number of sides on the polygon chosen, or n, meaning that patterns based on triangles will have three possible folding solutions, and so on. These factors combine to create a large design space to work with, as any unique pattern will generally have multiple viable circuits, and each circuit will have n possible folding sets.

Each folding set will result in a pair of panels on the top and a pair of panels on the bottom, with all other panels folded in between these two pairs. To find valid folding sets, a dyad of pairs which correspond to each other must first be found.

To find the folding sets and corresponding pairs which may fold with each other for each pattern\circuit combination, an initial fold line is chosen from the lines which intersect the circuit within the pattern. This line is then reflected over the subsequent fold lines in the Hamiltonian circuit found. This process is repeated until the current reflected line returns to the original fold line, as shown applied to an arbitrary pattern\circuit combination in Fig.4. This process gives both lines on the pattern that intersect with the circuit and are used as lines along which the pattern is folded, and lines which do not intersect with the pattern and used as lines on which the pattern will be cut. In Fig. 4, lines which result in cut lines are shown as dashed, and lines which result in fold lines are shown as solid lines. Panels adjacent to fold lines comprise a pair of a fold group. If the reflected line does not return to the original fold line after traversing the circuit, the circuit will not fold rigidly and may not be used to fold the thickened pattern. It should be noted that the circuit may be followed in either the clockwise or counter clockwise direction, as both will yield the same resulting pairs and cut lines.

This process is repeated *n* times, with the initial fold line for each folding set being chosen from the next line intersecting with the circuit which has not been included in a previous folding set. These fold groups may result in just one line which intersects the circuit, in which case they will not include a dyad of pairs and will not be able to fold. It can also be noted that several fold groups may share resulting cut lines during the reflection process; however, they are not part of the same fold groups because they result from reflections along different circuit lines and do not result in any interference with each other. Fig.5 shows how this process is repeated for each of the folding sets, and Fig.6 shows the final sets which intersect the folding joints on the pattern. Note that each color represents a unique folding set, with pairs connected by like colors representing valid dyads. When using patterns based on square grids, the same method is used; however, a simplified method for finding pairs can be used. This is explained and shown in Appendix 9.

Once the folding sets have been found, any pairs within the set may be used as a top or bottom dyad, as long as there is at least one other dyad contained within the set. For example, if there are 3 pairs found within a set, any two of them may be used to create the top and bottom dyads, with all other panels folded in between them.

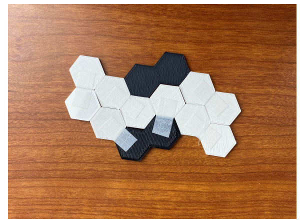
To compactly stow a pattern with the highest stowage efficiency, a pattern should have an equal number of panels in both the clockwise and counter-clockwise directions between the dyad of pairs, which will result in each column of the folded pattern having an equal number of panels. This is simple to determine, as the number of panels between each dyad can be counted, and the dyads with an even number of panels can be favored. In the case illustrated in Fig.6, the combination given by the top blue pair and the leftmost blue pair is only valid dyad of pairs which meets this criteria and would result in a perfectly compact stowed pattern. Other considerations will be discussed in Section 6 and will be shown applied to specific patterns.

4. PREDICTING FOLDING VALIDITY (MATH)

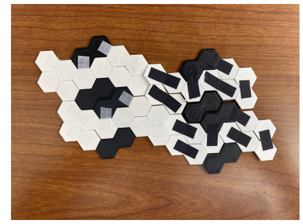
MATH DEPARTMENT CONTENT HERE

5. ADDITIONAL TECHNIQUES

There are several additional techniques which can be used to modify the performance and behavior of patterns designed using Hamiltonian circuits. These include tessellation, merging panels, and incorporating open loops, and additional techniques are sure to be discovered when applied to new use cases.



(a)



(b)

FIGURE 7: TEMPORARY FIGURE. A HEXAGONAL PATTERN IS USED TO ILLUSTRATE HOW THIS WORKS WITH ANYTHING. IT WOULD BE EASIER FOR SQUARES.

5.1 Tessellation

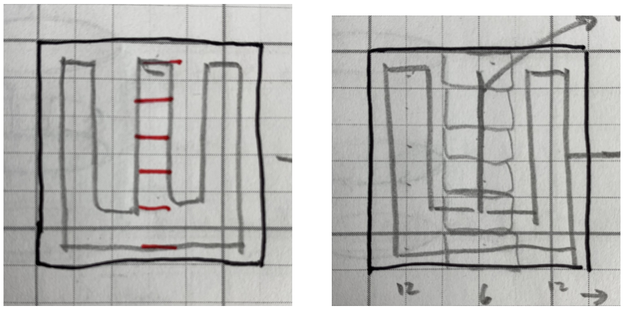
One of the benefits of patterns based on standard panels is that they can be easily modified and expanded. This allows for convenient tessellation of multiple patterns to produce large arrays over time from smaller sections. Yang et al. [6] demonstrated this concept using Resch patterns as a basis for folding thick-origami patterns, and Yang et al. [7] showed how clever tessellation can be used to reduce the degrees-of-freedom of patterns designed with Hamiltonian circuits. Facilitated tessellation is a property of these types of patterns and makes them a compelling candidate for future work in creating large arrays in space from the aggregation of a group of smaller payloads. This would also allow for the gradual growth of space-based systems, meaning that performance could increase with time as more units are added to the array, delaying the effects of age and wear on a system and increasing its lifetime.

Consider the example shown in Fig.7 as an array of solar cells. Fig.7 (a) shows a base unit made up of a symmetrical pattern based on hexagons, with an even number of panels between the top and bottom dyads. One its own, this pattern is a viable candidate that would stow panels of any uniform thickness compactly and deploy out of a large array. Fig.7 (b) shows how multiple instances of this base pattern can be combined to produce a larger array with a proportional increase in performance. Note that in this example, a hexagonal pattern is used for illustrative purposes to highlight the benefits of using a pattern based on unit cells, as it facilitates designs based on even more complex base polygons. Patterns based on square or rectangular cells lend themselves to easier folding and merging of panels, as well as easier tessellation.

5.2 Merging Panels and Incorporating Open Chains

One of the main drawbacks to the design of patterns using this methodology is that they result in long kinematic chains with many degrees-of-freedom. This can be mitigated by merging panels which experience concurrent motion in the folding sequence, as shown in the example in Fig.15. The effect of merging panels can be directly calculated using the Chebychev–Grübler–Kutzbach criterion, which can be used to find the mobility of both simple open and simple closed chains. The mobility of a simple open chain is given by

$$M = \sum_{i=1}^{j} (f_i) \tag{1}$$



(a) (b)

FIGURE 8: TEMPORARY FIGURE. CHANGE TO ILLUSTRATOR FIGURE.

and the mobility of a simple closed chain is given by

$$M = \sum_{i=1}^{j} (f_i - 6) \tag{2}$$

where M is the degrees-of-freedom of the system, j is the number of joints, and f_i is the freedom of each joint.

Merging panels should occur in every pattern in the dyad of pairs that define the circuit, as neither panel in a pair is moving relative to the other. The example in Fig.8 shows the difference that merging multiple panels can make to the mobility of the system. In Fig.8 (a), the circuit imposed on the 36 panel square pattern results in a mobility of $M = 30$, which is calculated using Eq. 2 for a single closed chain. As panels are merged, however, the circuit becomes an closed chain connected to an open chain. Fig.8 (b) shows this, and the resulting mobility can be calculated as $M = 24$, found by using both Eqs. 1 and 2 for each corresponding section of the new circuit. The effect of the creation of open chains while merging panels was originally noted by Yang et al.[3], and merging panels generally results in lowering the overall complexity of the system despite the new open chains.

Incorporating open chains can also be useful for achieving geometry which may not be able to fold as a closed circuit. Single closed-chain patterns may be impossible for a given grid due to a variety of reasons, such as having an odd number of panels or an even number of panels containing no pair dyads within a folding set. Nesting combinations of both open and closed chains allows for creation of deployed geometries which are otherwise impossible. To accommodate this geometry, panels can be removed to produce a pattern with an even number of panels, which is then used to find a valid folding pattern. Once this pattern is found, the removed panels can be added back to the pattern as an open chain, as long as it is connected to one of the top or bottom pairs. This results in a closed chain with $n - r$ panels and an open chain with $r + 1$ panels, where r is the number of panels which have been removed. An example of this is apparent in hexagonal patterns with a circular shape, such as those shown in Sections 6.4 and 6.5.

6. APPLICATIONS IN PATTERNS WITH POTENTIAL FOR DEPLOYABLE RF ARRAYS

The design space created by the Hamiltonian circuit methodology is large; however, when constrained by other factors, the design space shrinks considerably. This section will consider patterns which can be used for large (5-10 meter diameter), space-based antennas with RF applications, although other applications may result in differing ideal geometries, such as for SmallSat applications or arrays used primarily to generate solar power. Some constraints that this section considered were manufacturability, practicality, and RF applications. Practicality takes into account the total number of panels and degrees-of-freedom in the system. Because this work considers the application of an RF antenna, deployed areas closer to that of an inscribed circle were viewed as more ideal. Although every application is unique, this work considered large antenna applications and made the assumption that the largest panel would be on the scale of 1 meter, and as such sought to maintain a roughly 1:5-1:10 ratio between the width of one panel and the width of the entire array. Another consideration that was used throughout was that the pattern should use a folding circuit that resulted in an even number of panels in each stack, as to maximize volume efficiency, as explained in Section 2.

With each of these considerations in mind, the design space shrinks appreciably. This work will introduce 5 patterns that satisfy these requirements and are viable candidates for future work on large space-based antenna designs. Trade-offs for each option will be discussed. These designs use both square panel and hexagonal uniform panels. The overview of each pattern with its accompanying parameters can be seen in Fig.9, which also includes a scale comparison of the circumscribed circle area of the each deployed pattern, and a scale comparison of the stowed side and top area of each pattern, with associated parameters. Note that the pattern shown on the top row, far right of Fig.9 also uses equilateral triangles on each corner in additional to the primarily square grid. To compare each pattern, calculations were kept general, considering a unit panel with a length of a.

6.1 36 Panel Square Pattern

The 36 panel square pattern, shown in Fig.10, is the result of the most basic design from the requirements. It is intuitive and simple to draw. Additionally, there are a plethora of viable Hamiltonian circuits that may be created with this grid, including many that result in reduced degrees-of-freedom from the merging of panels which have identical kinematics. This pattern results in the highest overall area of all the patterns considered; however, it results in a significant amount of area which is unused in RF applications on each corner.

6.2 32 Panel Square Pattern

In exploring the 36 panel square pattern, it was found that by reducing the diameter of the inscribed circle slightly, the four corner panels could be removed, as shown in Fig.11. This results in an 11.76% decrease in total RF area, with an identical 11.76% decrease in the total mass and stowed volume of the antenna. This configuration is preferable to the 36 panel square pattern when mass and stowed volume are prioritized over pure performance. This design also maintains the use of solely square

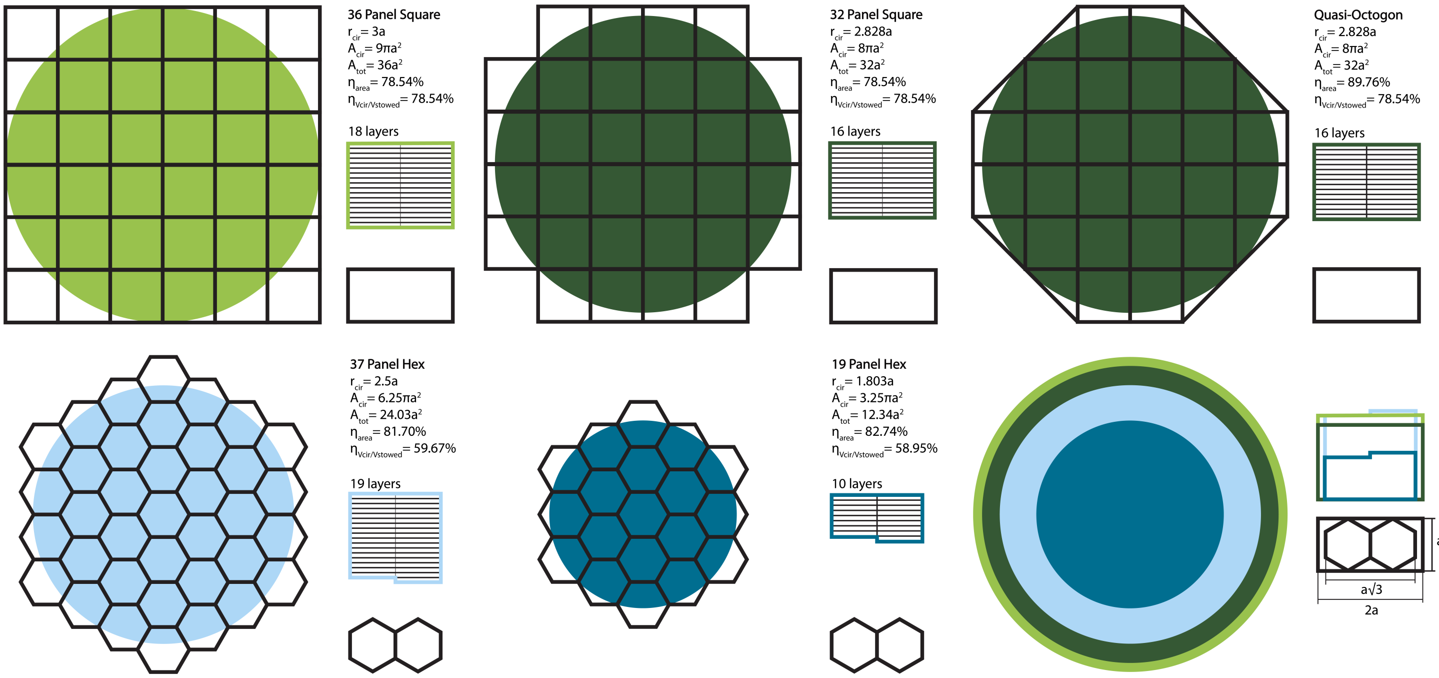


FIGURE 9: REMAKE, INCLUDE CIRCUIT. SUMMARY OF THE KEY PARAMETERS OF ALL EXPLORED PATTERNS. NOTE THAT THEY BOTTOM RIGHT SHOWS A SCALE COMPARISON OF THE CIRCUMSCRIBED CIRCLE AREA OF THE EACH DEPLOYED PATTERN ON THE LEFT, AND A SCALE COMPARISON OF THE STOWED SIDE AND TOP AREA OF EACH PATTERN ON THE RIGHT, WITH ASSOCIATED PARAMETERS. DARK BLUE CORRESPONDS WITH THE 19 PANEL HEXAGONAL PATTERN, LIGHT BLUE CORRESPONDS WITH THE 37 PANEL HEXAGONAL PATTERN, DARK GREEN CORRESPONDS WITH THE QUASI-OCTAGON AND 32 PANEL SQUARE PATTERNS, AND LIGHT GREEN CORRESPONDS WITH THE 36 PANEL SQUARE PATTERN.

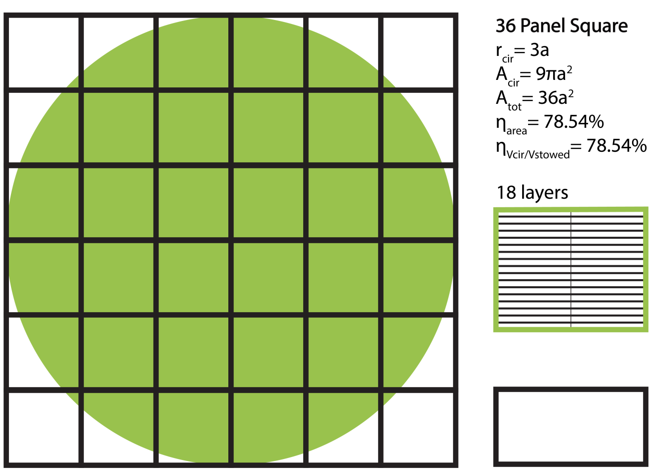


FIGURE 10: 36 PANEL SQUARE PATTERN WITH ASSOCIATED PARAMETERS. CIRCUMSCRIBED CIRCLE FOR RF USE SHOWN IN LIGHT GREEN. SIDE AND TOP VIEW OF STOWED PATTERN IS SHOWN ON RIGHT.

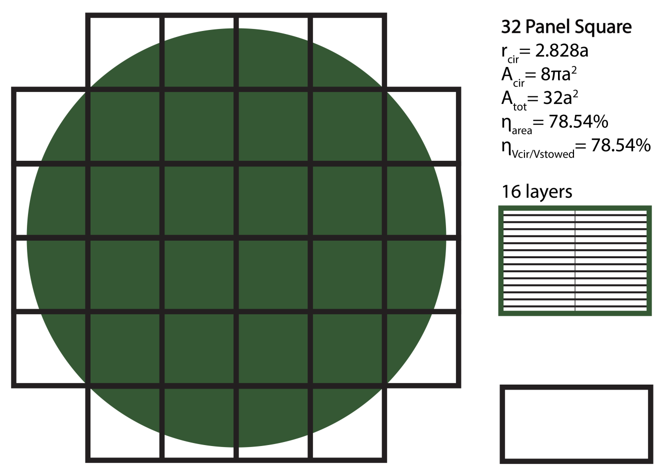


FIGURE 11: 32 PANEL SQUARE PATTERN WITH ASSOCIATED PARAMETERS. CIRCUMSCRIBED CIRCLE FOR RF USE SHOWN IN DARK GREEN. SIDE AND TOP VIEW OF STOWED PATTERN IS SHOWN ON RIGHT.

panels, simplifying manufacturing and the incorporation of existing components.

6.3 Quasi-Octagon Square Pattern

The benefits of the 32 panel square pattern in mass reduction can be further improved by cutting off the outer halves of the corner panels, as shown in Fig.12. This results in a 25% decrease in total mass as compared to the 36 panel square pattern, while retaining the same 11.76% decrease in total usable RF area. This pattern maintains the same stowed volume as the previous 32 panel square pattern and increases the number of unique panels, but it would be preferable when mass must be minimized as much as possible.

It should be noted that in continuing the pattern of optimization for reduced mass, all excess which did not conform to the circumscribed circle could be removed; however, as shown with

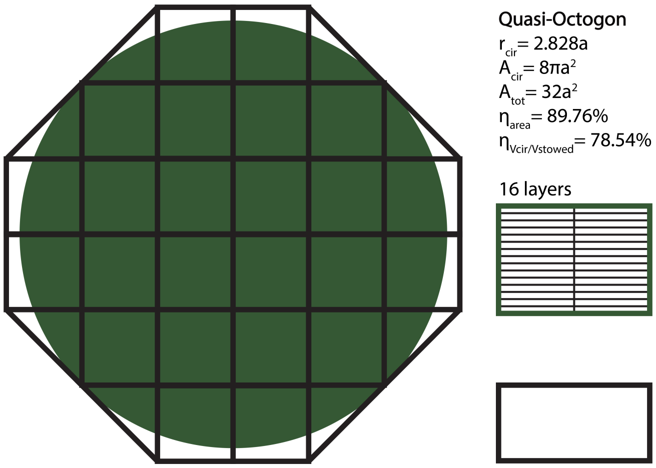


FIGURE 12: QUASI-OCTAGON SQUARE PATTERN WITH ASSOCIATED PARAMETERS. CIRCUMSCRIBED CIRCLE FOR RF USE SHOWN IN DARK GREEN. SIDE AND TOP VIEW OF STOWED PATTERN IS SHOWN ON RIGHT.

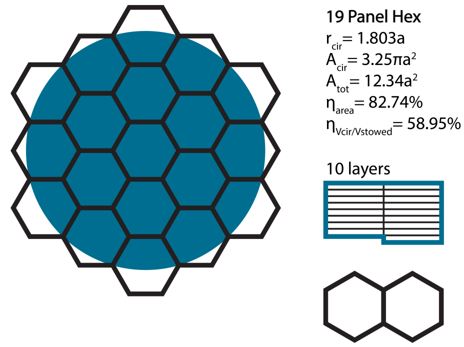


FIGURE 13: 19 PANEL HEXAGON PATTERN WITH ASSOCIATED PARAMETERS. CIRCUMSCRIBED CIRCLE FOR RF USE SHOWN IN DARK BLUE. SIDE AND TOP VIEW OF STOWED PATTERN IS SHOWN ON RIGHT. NOTE THAT BECAUSE PATTERN CONTAINS AN ODD NUMBER OF PANELS AND MUST BE MODIFIED TO INCLUDE AN OPEN CHAIN, THE RIGHT SIDE OF THE STOWED PATTERN HAS ONE MORE PANEL THAN THE LEFT SIDE.

the Quasi-Octagon pattern, the stowed volume remains the same because the number of panels and corresponding thickness when stowed is unchanged, and the mass gains from further material removal would be minimal, decreasing at most by 10.24% while increasing the manufacturing complexity of the pattern by several unique panels. Additionally, a similar pattern could be formed with smaller and smaller panels, allowing for more panels to be removed from each corner as to increase the area efficiency of the design, but such a design would be increasingly impractical, as the number of panels required would increase exponentially. As such, the Quasi-Octagon pattern is considered in this work to be the most optimal version of minimizing mass, maximizing RF area, and maintaining practicality for the purposes of manufacturability and deployment by modifying the initial panel shape.

6.4 19 Panel Hexagonal Pattern

When considering the RF requirement that the deployed area would be evaluated on its conformance to a circular shape, patterns based on hexagonal grids were a natural solution. Two patterns based on hexagons were explored, one with 19 panels, shown in Fig.13, and one with 37 panels, shown in Fig.14.

By using a grid of hexagons, which are by their nature more circular than squares, the 19 panel pattern is able to achieve a used area efficiency 82.74%, a 5.21% increase from the 36 panel square and 32 panel square patterns. This pattern has as significantly smaller overall area than other patterns, but this is due to the use of a unit cell measurement, and as such, other metrics, such as deployed area efficiency and stowed volume efficiency may be considered more useful for comparison purposes.

The 19 panel hexagonal pattern also shows an interesting consideration when using hexagonal patterns with an aspect ratio of 1, which is that such patterns have an odd number of panels which are unable to evenly stack in two piles. Because of this, one panel is removed from the pattern when determining Hamiltonian circuits, using the method described in Section 5.2.

6.5 37 Panel Hexagon Pattern

The 37 panel hexagon pattern takes the benefits of the 19 panel hexagon pattern and increases the relative RF area by adding an order to the outside of the pattern, which is shown in Fig.14. Implementing the technique of trimming panel shapes to reduce the mass of the system and maintain the viable RF area as shown with the Quasi-Octagon pattern was considered, and it was found that removing the outer half of the most unused hexagons would increase the area efficiency of the pattern by 7.21% to a total of 88.91% while only increasing the complexity by one unique panel. This would result in a very good used area efficiency, second only to the Quasi-Octagon pattern. However, this idea was overshadowed by the fact that hexagonal patterns are much more complex to fold, and the kinematics of which are difficult to predict and simplify. In fact, in exploring many hexagon based patterns, very few were found to have any folding sequences that could be simplified in any way through the merging of panels, and often were subject to a unique series of steps involving non-adjacent panels, making if very difficult to predict valid deployment sequences. These factors negate the benefits of compact stowage, in that they would generally require complex deployment mechanisms to be incorporated into the design. Because of this, hexagonal patterns were not explored further, and it is put forward that for the application of this section, they are not ideal candidates for deployable arrays which are low-mass, high-area, and practical.

6.6 Comparison of Potential Patterns

When considering which pattern may be the best fit for a given application, it is appropriate to compare the patterns based on a variety of criteria. Deployed area and stowed volume are often used as the most basic criteria for deployable space-based applications, and variations of these are used in this work. Additionally, prototypes from various patterns were tested and results will be discussed, although not all models will be shown in this work.

Various advantages and disadvantages of each pattern have been discussed and will be used to make determinations about

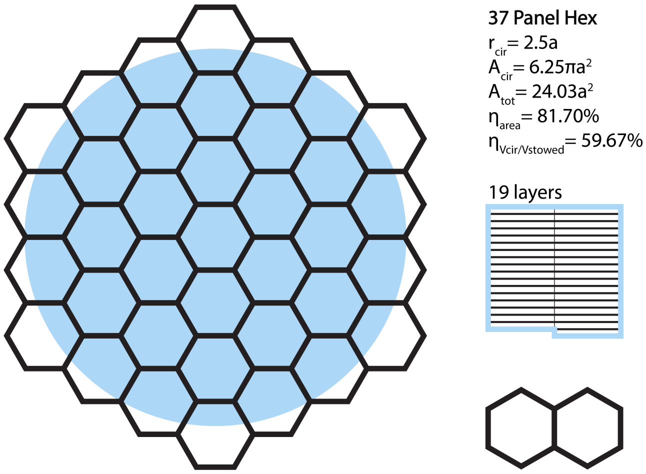


FIGURE 14: 37 PANEL HEXAGON PATTERN WITH ASSOCIATED PARAMETERS. CIRCUMSCRIBED CIRCLE FOR RF USE SHOWN IN LIGHT BLUE. SIDE AND TOP VIEW OF STOWED PATTERN IS SHOWN ON RIGHT. NOTE THAT BECAUSE PATTERN CONTAINS AN ODD NUMBER OF PANELS AND MUST BE MODIFIED TO INCLUDE AN OPEN CHAIN, THE RIGHT SIDE OF THE STOWED PATTERN HAS ONE MORE PANEL THAN THE LEFT SIDE.

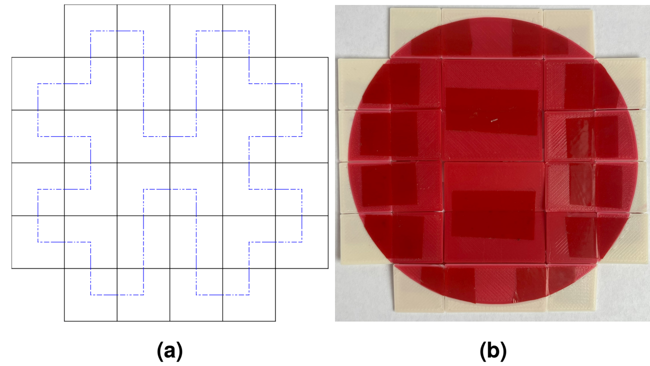


FIGURE 15: REMAKE AND USE OPEN CIRCUIT. EXAMPLE IMPLEMENTATION. A) VALID HAMILTONIAN CIRCUIT FOR 32 PANEL SQUARE GRID. B) 3D PRINTED PROTOTYPE SHOWING EXAMPLE OF MERGED PANELS TO REDUCE THE OVERALL DEGREES-OF-FREEDOM.

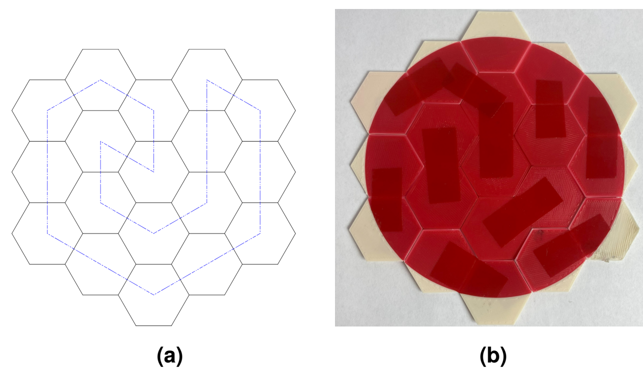


FIGURE 16: UPDATE AND USE OPEN CIRCUIT. EXAMPLE IMPLEMENTATION. NOTE THAT BECAUSE THE GRID HAS AN ODD NUMBER OF PANELS, ONE PANEL MUST BE CONNECTED BY A SEPARATE CIRCUIT ON THE BACK AND IS NOT SHOWN IN THE HAMILTONIAN CIRCUIT. A) VALID HAMILTONIAN CIRCUIT FOR 18 PANEL HEXAGONAL GRID. B) 3D PRINTED PROTOTYPE.

the feasibility and attractiveness of each design. A general theme that was found was that patterns based on a square grid are more likely to be able to be simplified by finding methods of concurrent deployment between different panels to reduce the degrees-of-freedom of the system. Contrarily, patterns based on hexagonal grids resulted in complicated kinematics which are non-trivial and are rarely, if ever, able to be simplified. An example prototype of the 32 panel square pattern is shown in Fig.15 (b), with its corresponding Hamiltonian circuit shown in Fig.15 (a). This prototype shows an example of a simplified pattern, with the vertically middle two columns shown with “merged" panels. An example of the 19 panel hexagonal pattern is shown in Fig.16 (b), with its corresponding Hamiltonian circuit shown in Fig.16 (a).

In general, this makes square-based patterns more compelling candidates for design than their hexagonal counterparts. This is especially highlighted by the Quasi-Octagon design, which incorporates the simplicity of the square grid with an area efficiency exceeding that of the hexagonal grids. Because the primary motivation and advantage of the hexagonal based patterns was their area efficiency, which was superior when compared to the 36 and 32 panel square patterns, this advantage in the Quasi-Octagon pattern suggests that patterns based on hexagonal grids are not worthy of further exploration for the purposes of designing simple, large, space-based deployable arrays. Even if there were significant stowed volume or deployed area efficiency advantages with hexagonal patterns, the complicated nature of the resulting kinematics alone would be enough to give any designer pause. One of the aims of this work was to explore patterns based on Hamiltonian circuits and justify why some patterns are good and why some patterns are not suitable for the intended applications. Patterns based on hexagons are too complicated, unconstrained, and unpredictable to be of use in large applications, and there exist other options which have the same benefits and are far better suited to these uses.

One method of determining which design is the best for a given application is to use a weighted ranking method based on relevant design parameters and characteristics, such as stowed volume, total mass, and deployed area efficiency. A summary of relevant characteristics for the patterns discussed is given in Table 1. An example of a weighted ranking system approach to determining which pattern is best suited to the given use case is shown in Table 2. In this example, the best pattern for each major parameter received 2 points, the second best received 1 point, and the worst received -1 point. While this simple example has limitations, it is sufficient to illustrate the method. In this example, it can be seen that several patterns, such as the 36 and 32 panel square patterns, are the best in some categories (deployed area and volume efficiency) but are the worst in deployed area efficiency. The best pattern overall was found to be the Quasi-Octagon, as it combines the best volume efficiency of the square patterns with the best area efficiency. Another way to further examine various patterns would be to apply a weight to each individual parameter, such as if volume efficiency is more important than deployed area efficiency, and future designers would be wise to consider their needs when making a decision as to which pattern is best suited to their application.

7. FUTURE WORK

One of the initial considerations used in this work when exploring potential patterns was that each stack in the stowed patterns should have the same number of panels. While this simplifies the stowed volume, it is not strictly necessary. One of the major benefits of using Hamiltonian circuits is that they have a simple thickness accommodation built-in, meaning that thick panels do not complicate the pattern implementation. This means that patterns which have an unequal number of panels in each stack could be utilized in other ways, such as thickening panels in the stack with fewer panels, and incorporating electronics or other components into them. This has the potential of increasing the volume efficiency of the entire satellite system as a whole, rather than just considering the antenna alone. Ideally, every component in the satellite could be incorporated into a thick panel with RF on the outside, and the entire system could fold compactly and deploy as one unit. Each stowed pattern resulted in a shape that was twice the size of each individual panel. Future work could also modify this, and create a pattern using a grid of “half-squares" or “half-hexagons", such that the final stowed shape was a single unit. This would be functionally the same process of design, and would result in twice as many panels and degrees-of-freedom, but could result in compact geometries for specific applications.

This work also limited its study to that of single antenna applications. Another interesting benefit of patterns based on Hamiltonian circuits is that they may be easily tessellated, as discussed in Section 5.1. Future work may benefit from exploring patterns which are especially designed to be tessellated once deployed in space.

8. CONCLUSION

The design space created by Hamiltonian Circuits as an origin for creating novel origami patterns yields exciting results and has immense potential for unique deployable applications. The designs that result from this methodology have simple thickness accommodation, high stowage efficiency, and predictable deployed area efficiency. This makes them prime candidates for future exploration and implementation, which is certain to give promising results and functional designs.

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TABLE 1: SUMMARY OF PATTERN TRADEOFFS

Metric	36 Panel, Square	32 Panel, Square	Quasi-Octagon	19 Panel, Hexagonal	37 Panel, Hexagonal
Number of Panels	36	32	32	19	37
Degrees-of-Freedom	29	25	25	19	37
A_{panel}	a^2	a^2	$a^2 and\ \frac{1}{2}\ a^2$	$\frac{3\sqrt{3}}{8}a^2$	$\frac{3\sqrt{3}}{8}\ a^2$
A_{total}	$36a^2$	$32a^2$	$28a^2$	$\frac{57\sqrt{3}}{8}\ a^2 \approx 12.34a^2$	$\frac{111\sqrt{3}}{8}\ a^2 \approx 24.03a^2$
$r_{circumscribed}$	$3a$	$2a\sqrt{2}$	$2a\sqrt{2}$	$\frac{\sqrt{52}}{4}a$	$\frac{\sqrt{5}}{2}a$
A_{RF}	$9\pi a^2$	$8\pi a^2$	$8\pi a^2$	$\frac{52}{16}\pi\ a^2$	$\frac{25}{4}\pi\ a^2$
$A_{RF}(\#)$	28.27	25.13	25.13	10.21	19.64
η_A	$\frac{\pi}{4} \approx 78.54\%$	$\frac{\pi}{4} \approx 78.54\%$	$\frac{2\pi}{7} \approx 89.76\%$	$\frac{26\pi}{57\sqrt{3}} \approx 82.74\%$	$\frac{50\pi}{111\sqrt{3}} \approx 81.70\%$
Thickness stacked	$18t$	$16t$	$16t$	$10t$	$19t$
Length	$2a$	$2a$	$2a$	$a\sqrt{3}$	$a\sqrt{3}$
V_{stowed}	$36ta^2$	$32ta^2$	$32ta^2$	$10\sqrt{3}ta^2 \approx 17.3ta^2$	$19\sqrt{3}ta^2 \approx 32.9ta^2$
η_V	$\frac{\pi}{4} \approx 78.54\%$	$\frac{\pi}{4} \approx 78.54\%$	$\frac{\pi}{4} \approx 78.54\%$	$\frac{52\pi}{160\sqrt{3}} \approx 58.95\%$	$\frac{25\pi}{76\sqrt{3}} \approx 59.66\%$
η_{cuboid}	100%	100%	100%	75%	75%
η_{panel}	3.54%	3.98%	3.98%	6.36%	3.31%

TABLE 2: EXAMPLE WEIGHTED RANKING METHOD FOR DETERMINING PREFERRED PATTERN

Metric	36 Panel, Square	32 Panel, Square	Quasi-Octagon	19 Panel, Hexagonal	37 Panel, Hexagonal
$r_{circumscribed}$	3	2.828	2.828	1.803	2.5
A_{RF}	28.27	25.13	25.13	10.21	19.64
η_A	78.54	78.54	89.76	82.74	81.70
Thickness stacked	18	16	16	10	19
Length	2	2	2	1.732	1.732
V_{stowed}	36	32	32	17.3	32.9
η_V	78.54	78.54	78.54	58.95	59.66
η_{cuboid}	100	100	100	75	75
η_{panel}	3.54	3.98	3.98	6.36	3.31
Best (2)	2	1	2	1	0
Second best (1)	0	2	2	1	0
Worst (-1)	1	1	0	2	2
Score	3	3	6	1	-2

REFERENCES

[1] R. J. Lang, K. A. Tolman, E. B. Crampton, S. P. Magleby, and L. L. Howell, "A review of thickness-accommodation techniques in origami-inspired engineering," *Applied Mechanics Reviews*, vol. 70, no. 1, 2018.

[2] *A Preliminary Approach to Select an Origami Source Pattern for Deployable Space Arrays*, ser. International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, vol. Volume 7: 46th Mechanisms and Robotics Conference (MR), 08 2022. [Online]. Available: https://doi.org/10.1115/DETC2022-90033

[3] J. Yang, Y. Zhang, M. N. Chatzis, and Z. You, "Folding and deploying identical thick panels with spring-loaded hinges," *Extreme Mechanics Letters*, vol. 52, p. 101637, 2022. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S2352431622000219

[4] ——, "Data for "folding and deploying identical thick panels with spring-loaded hinges"," *Data in Brief*, vol. 43, p. 108388, 2022. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S2352340922005856

[5] *Compactly Folding Rigid Panels With Uniform Thickness Through Origami and Kirigami*, ser. International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, vol. Volume 5B: 43rd Mechanisms and Robotics Conference, 08 2019. [Online]. Available: https://doi.org/10.1115/DETC2019-97946

[6] F. Yang, M. Zhang, J. Ma, Z. You, Y. Yu, Y. Chen, and G. H. Paulino, "Design of single degree-of-freedom triangular resch patterns with thick-panel origami," *Mechanism and Machine Theory*, vol. 169, p. 104650, 2022. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0094114X21003815

[7] J. Yang, X. Zhang, Y. Chen, and Z. You, "Folding arrays of uniform-thickness panels to compact bundles with a single degree of freedom," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 478, no. 2261, p. 20220043, 2022. [Online]. Available: https://royalsocietypublishing.org/doi/abs/10.1098/rspa.2022.0043

[8] W. Hamilton, *The Mathematical Papers of Sir William Rowan Hamilton*. The University Press, 2006. [Online]. Available: https://books.google.com/books?id=-vI8AAAAIAAJ

9. APPENDIX I: SIMPLIFIED FOLDING MODEL FOR SQUARE-BASED PATTERNS

Talk about and show figures for the simplified method of folding squares. Include explanation of why it works, 90 degree angles etc.